Application of ARMAV for Modal Identification of the Emerson Bridge

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ABSTRACT: Modal identification is performed on measured data from the well known cable-stayed bridge (Bill Emerson Memorial Bridge). The ARMAV technique is employed to analyze the recorded ambient vibration data from sixteen of the eighty-four channels of acceleration from the bridge. To validate the results, the modal identification results obtained through ARMAV are compared to the modal parameters from an existing (without updating methods) finite element model of the bridge built based on the drawings. Five vertical vibrational modes below 1 Hz are identified and validated through comparison with the model.

1 INTRODUCTION

Large flexible structures such as cable-stayed bridges sustain a crucial role in transportation networks. Working as the nodes in the network, the failure of any one of these structures can significantly impact the economy of a region or a country. Currently the prevailing method for structural evaluation is simple visual inspection. However, these conventional techniques are not able to achieve a high level of reliability or efficiency. Factors such as fear of traffic, visual acuity, light intensity, inspector workload, perceptions of maintenance, complexity, and accessibility resulted in significant variability in these routine visual inspections.

To provide an accurate estimate of structural damage, the reliable identification of modal properties is a prerequisite. Although forced vibrations provide accurate quantitative modal information, the use of ambient loading constitutes an attractive alternative in terms of cost and simplicity. The auto-regressive and moving average vector (ARMAV) technique is one of the most promising techniques to make use of ambient vibration data (Giraldo et al. 2006). By means of ARMAV technique, modal analysis can be conducted for structures under unknown excitation forces, presumed to be random, such as wind gusts and traffic loads, which allow the fully automated real-time monitoring of the structure under in-service damage assessment.

The purpose of this paper is to present preliminary results regarding the modal identification of the Bill Emerson Memorial Bridge using ambient vibration response data. The Emerson Bridge in Cape Girardeau, Missouri is one of a small number of structures that are instrumented in the US. The instrumentation installed on the bridge and surrounding soil was included to evaluate structural behavior and seismic risk (Çelebi 1999). The availability of the recorded data from the bridge presents an excellent opportunity to examine the capabilities of several of the modal identification techniques for structures in this class. The presence of coupled motions and closely-spaced modes presents challenges to the researcher and engineer for these structures. In this preliminary study, we consider vertical modes of the structure. ARMAV is applied to identify vertical modes below 1 Hz. The implementation of the ARMAV approach is considered by examining the accuracy of the results using various orders of models. The results demonstrate that this approach is promising for automated modal identification of this structure.

2 THE BILL EMERSON MEMORIAL BRIDGE

2.1 *Description of the Bridge and Instrumentation*

The cable-stayed bridge data considered in this paper was measured from the Emerson Bridge spanning the Mississippi River (between Missouri 74–Illinois 146) near Cape Girardeau, Missouri. The bridge was designed by the HNTB Corporation (Hague 1997). Seismic considerations were strongly considered in the design of this bridge due to the location of the bridge (the New Madrid seismic zone) and its critical role as a principal crossing of the Mississippi River. As shown in Figure 1, the bridge is composed of two towers, 128 cables, and 12 additional piers in the approach bridge from the Illinois side (length 1205.8 m). The main span is 350.6 m in length, the side spans are 142.7 m in length, and the approach on the Illinois side is 570 m.

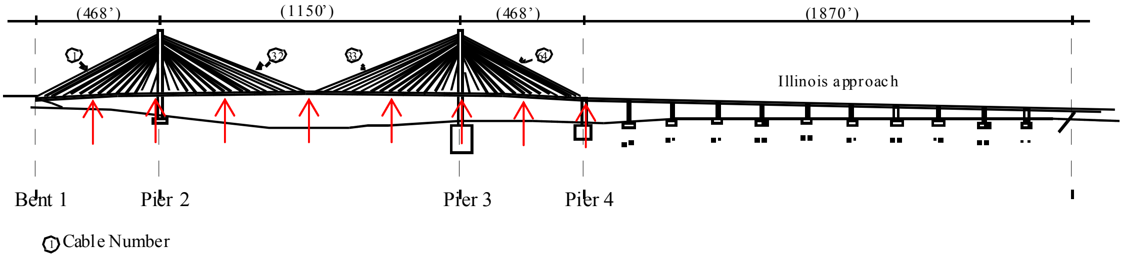


Figure 1. Sensor placement on Emerson Bridge (Arrows denote the sensors)

Sixteen 6.67 MN (1,500 kip) shock transmission devices are employed in the connection between the tower and the deck to allow for expansion of the deck due to temperature changes. Additionally, earthquake restrainers are employed in the transverse direction at the connection between the tower and the deck, and the deck is restrained in the vertical direction at the towers. The bearings at bent 1 and pier 4 are designed to permit longitudinal displacement and rotation about the transverse and vertical axis.

There are a total of 84 accelerometers installed on the bridge and in the surrounding soil. As shown by the arrows in Figure 1, in the modal analysis performed in this paper, only 16 vertical motion signals on the bridge deck were used. Each arrow represents two channels, which are located on the two edges of the deck.

2.2 *Construction of the Finite Element Model*

A finite element model (FEM) of the Emerson Bridge was constructed for the benchmark problem on structural control of seismically excited cable-stayed bridges (Dyke et al. 2003, Caicedo et al. 2002). The model is shown in Figure 4 and has a total of 579 nodes, 420 rigid links, 162 beam elements, 134 nodal masses and 128 cable elements (see Fig. 2). The towers are modeled using 50 nodes, 43 beam elements and 74 rigid links. Constraints are applied to restrict the deck from moving laterally at piers 2, 3 and 4. Boundary conditions restrict the motion at bent 1 to allow longitudinal displacement (X) and rotations about the Y and Z axes. The cables are modeled with truss elements. In the FEM the nominal tension is assigned to each cable.

Cable-stayed bridges exhibit nonlinear behavior due to variations of the catenary shape of the inclined cables, cable tensions that induce compression forces in the deck and towers, and large displacements. A nonlinear static analysis was performed using the commercial finite element program ABAQUS®, giving the model tangent stiffness matrix at the (deformed) equilibrium position. In ABAQUS®, the B31 beam element was used for the structural beam element, and the element T3D2 was used for the cable elements.

The catenary shape and its variation with the axial force in the cable are modeled using an equivalent elastic modulus (Ernst 1965). The cable elements are modeled as truss elements in ABAQUS®, and their equivalent elastic moduli are used in the nonlinear static analysis. The deck is comprised of two main steel girders along each longitudinal edge of the deck supporting the concrete slab. The deck was modeled by the method described by Wilson & Gravelle (1991), and is treated as a C-shaped section.

A full description of the FEM is provided in Dyke et al. (2003) and Caicedo et al. (2002).

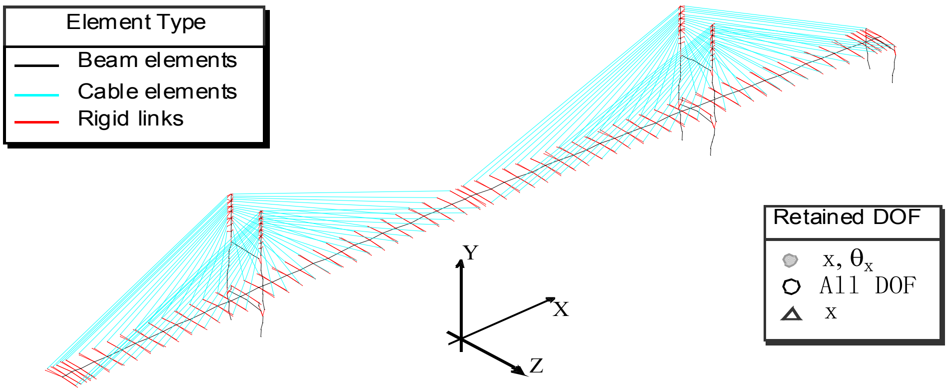


Figure 2. Finite Element Model of the cable-stayed bridge

3 MODAL IDENTIFICATION TECHNIQUE

In this paper, ARMAV is used to identify the modal parameters. The ARMAV model has become a standard tool in control and econometrics for both system description and control design. Several algorithms have been proposed to implement prediction error methods, such as Prediction error method-least squares estimation and maximum likelihood estimation (MLE). In this study, the prediction error method is carried out using a two-stage least squares approach (PEM/LS, see Ljung 1987). The reason for this choice is that the PEM for Gaussian distributed prediction errors is asymptotically unbiased and efficient. Further, the use of the PEM enables an estimate of the associated uncertainties of the estimated modal parameters.

As the starting step, a long auto-regressive (AR) model is fitted to the measurements using least squares (Andersen 1997). The auto-regressive model is of the form

$$\hat{y}(k) = -A_1 y(k-1) - A_2 y(k-2) - \ldots - A_n y(k-n) \tag{1}$$

where $\hat{y}(\bullet)$ and $y(\bullet)$ are the predicted and true outputs of the system, and A_i is the i-th auto-regressive coefficient of the n-order AR model, which has size $m \times m$ (m is the number of channels). Because the output $y(k)$ at time k cannot be determined exactly from the available data up to time $k-1$, it makes sense to determine the model parameter vector, defined as

$$\theta = \begin{bmatrix} A_1 & A_2 & \cdots & A_n \end{bmatrix}^T \tag{2}$$

so that the prediction error, defined as

$$\varepsilon(k,\theta) = y(k) - \hat{y}(k \mid k-1;\theta) \tag{3}$$

is minimized. $\hat{y}(k \mid k-1;\theta)$ is the predicted response at time k based on the parameters θ, and provided the available data up to time $k-1$.

In the case of multiple channels, the auto-regressive coefficients can be estimated as (Giraldo et al. 2006)

$$\begin{bmatrix} -A_1^T \\ -A_2^T \\ \vdots \\ -A_n^T \end{bmatrix} = \begin{bmatrix} y^T(n) & y^T(n-1) & \cdots & y^T(1) \\ y^T(n+1) & y^T(n) & \cdots & y^T(2) \\ \vdots & \vdots & \ddots & \vdots \\ y^T(j-1) & y^T(j-2) & \cdots & y^T(j-n) \end{bmatrix}^+ \begin{bmatrix} y^T(n+1) \\ y^T(n+2) \\ \vdots \\ y^T(j) \end{bmatrix} \tag{4}$$

where j is the length of the recorded measurements and '+' represents the pseudo inverse. The error of this AR model can be calculated by Eq. (3), i.e.

$$e(k) = \varepsilon(k,\theta) \tag{5}$$

In the second step, a pseudo-ARX (auto-regressive with exogenous input) model is estimated by using the error obtained from the AR model as the pseudo-input. This $\mathrm{ARX}(\mathrm{na},\mathrm{nc})$ model takes the form

$$\hat{y}(k) = -A_1 y(k-1) - A_2 y(k-2) - \ldots - A_{na} y(k-na) + C_1 e(k-1) + C_2 e(k-2) + \ldots + C_{nc} e(k-nc) \tag{6}$$

where na and nc are the orders for auto-regressive and moving average terms, respectively. The coefficients can be determined using the same least squares approach

$$\begin{bmatrix} -A_1^T \\ -A_2^T \\ \vdots \\ -A_{na}^T \\ -C_1^T \\ -C_2^T \\ \vdots \\ -C_{nc}^T \end{bmatrix} = \begin{bmatrix} y^T(na) & y^T(na-1) & \cdots & y^T(1) & e^T(na) & e^T(na-1) & \cdots e^T(na-nc+1) \\ y^T(na+1) & y^T(na) & \cdots & y^T(2) & e^T(na+1) & e^T(na) & \cdots e^T(na-nc+2) \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots \quad \vdots \\ y^T(j-1) & y^T(j-2) & \cdots y^T(j-na) & & e^T(j-1) & e^T(j-2) & \cdots \quad e^T(j-nc) \end{bmatrix}^+ \cdot \begin{bmatrix} y^T(na+1) \\ y^T(na+2) \\ \vdots \\ y^T(j) \end{bmatrix} \tag{7}$$

Finally, the system and output matrices of a state space realization associated with this ARMAV model of order (na , nc) can be formed by the state space system

$$A = \begin{bmatrix} 0 & I & 0 & \cdots & 0 \\ 0 & 0 & I & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & I \\ -A_{na} & -A_{na-1} & -A_{na-2} & \cdots & -A_1 \end{bmatrix} \text{ and } C = \begin{bmatrix} I & 0 & \cdots & 0 \end{bmatrix},$$

The natural frequencies f_r and damping ratios ς_r can be extracted from the eigenvalues λ_r of the AR matrix A as follows,

$$f_r = \left|\ln(\lambda_r)\right| \cdot f_s \tag{8}$$

$$\varsigma_r = \frac{-\operatorname{Re}\left[\ln(\lambda_r)\right]}{\left|\ln(\lambda_r)\right|} \tag{9}$$

with $r = 1, \ldots, m \cdot na$ and f_s is the sampling frequency. Let us define L , as the matrix of eigenvectors of A . The complex mode shapes of the output Φ are extracted from the matrix L as,

$$\Phi = C \cdot L \tag{10}$$

4 IMPLEMENTATION OF THE METHOD

The sizes of the system and output matrices of the estimated realization are significantly larger than those estimated with other techniques. The model characterized by an over-determination of the number of poles (the number of poles is higher than the number of structural poles) (Bodeux & Golinval 2001). As a result, along with the desired modal properties, several computational modal parameters are estimated. This phenomenon constitutes the main drawback in the use of ARMAV models for modal identification. To overcome this problem, the use of stabilization diagrams is helpful to separate true structural modes from non-physical ones. However, in this case, several models of different order have to be estimated. The application of the whole process is discussed subsequently.

4.1 *Data Processing*

The original data is sampled at the frequency 200 Hz. For each of the channels used herein, 30-min of data, or 360000 data points, was considered for this study. Because the modal analysis to be performed is concentrated on the low frequency range, before input the program with the data, a low-pass filter is first applied to remove the high frequency content in the signal and improve the accuracy of the modal analysis. Based on our previous experience with cable-stayed bridges and the preliminary information available through the FEM, the cut-off frequency of the filter is selected to be 5 Hz, which results in 9000 points per channel.

4.2 *Systematic Elimination of Modal Properties*

General knowledge of the system is fundamental for the identification of its dynamic properties. In cable-stayed bridges such as that considered herein, damping in all modes with consideration to soil-structure interaction is probably less than 3%. Recall that these are for small (ambient) motions about the equilibrium position of the structure, and therefore focus mostly on linear behavior. To eliminate some of the non-physical modes, it makes sense to set a threshold beyond which modes are not considered true physical modes and can be discarded. This threshold was set to 10% in this study. This threshold is useful for the ARMAV technique, eliminating some of the non-physical modes that, by nature, the algorithm estimates. Moreover, since the sampling frequency has been modified to 5 Hz, only the modes with frequencies below 2.5 Hz were potential modes resulting from the identification algorithm. This approach is common in experimental modal identification, as natural frequencies can easily be identified beforehand with frequency-domain techniques (e.g. power spectral density functions).

4.3 *Automatic Recognition of Similar Modes (Stability Diagram)*

As mentioned previously, with the ARMAV approach the resulting dynamic system overestimates the number of structural modes in the system. Distinguishing the physical modes of the real structural system from the non-physical modes is essential for system identification and challenging due to the presence of closely-spaced modes. When identifying the modal properties of dynamic systems, the user usually finds himself or herself making use of the same measurements repeatedly, while changing the algorithm parameters (e.g. ARMAV model change the orders for AR and MA parts). This practice results in multiple sets of modal properties that include the physical vibrational modes of the structure as well as modes due to noise. To determine the true modes, researchers have used the so-called stabilization diagrams for years. By plotting these diagrams, the user can easily visualize the frequencies that have been detected in a consistent manner.

The automatization of this process is somewhat complicated as many factors are involved. Basically, the detection of similar modes is based on two criteria, namely, 1) the proximity of the frequencies and, 2) the correlation of the relevant mode shapes. The first criterion was implemented by setting a 2% threshold; two frequencies are considered similar if their values are within 2% of each other. The correlation of the mode shapes was evaluated by calculating the modal assurance criterion (MAC) of the two vectors. Here, two mode shapes are considered similar if the MAC value is higher than 0.9. Two modes are considered similar if both criteria are met. “Stable” modes are determined by searching, in descending order, those modes with the most similar values. Moreover, during the development of a ‘Stability Diagram’, all the modal parameters (natural frequencies, damping ratios and mode shapes) are recalculated based on the average value over each ‘stable’ mode set.

4.4 *Accuracy of the ARMAV Model*

For ARMAV model, the best model order is in general not known, and several criteria have been proposed to evaluate the best model order (Bodeux & Golinval 2001). Two of the most widely used techniques for selecting the order of a parametric model are Akaike’s final prediction error criterion (FPE) and Akaike’s information theoretic criterion (AIC). These criteria are based on monitoring the decrease in the criterion function $V_N(\theta)$ as the order (na, nc) increases. The definition of $V_N(\theta)$ is,

$$V_N(\theta) = \det\left(\frac{1}{N}\sum_{n=1}^{N} \varepsilon[n \mid \theta] \cdot \varepsilon[n \mid \theta]^T\right) \tag{11}$$

where $\varepsilon(n,\theta)$ is the prediction error defined in Eq. (3). Here both the FPE and AIC criteria are employed to consider how the accuracy of the ARMAV model varies with the order (na,nc). The expressions for the FPE and AIC take the form

$$\text{FPE} = V_N(\theta)\frac{1+\upsilon/N}{1+\upsilon/N} \text{ and } \text{AIC} = N\log[V_N(\theta)] + 2\upsilon,$$

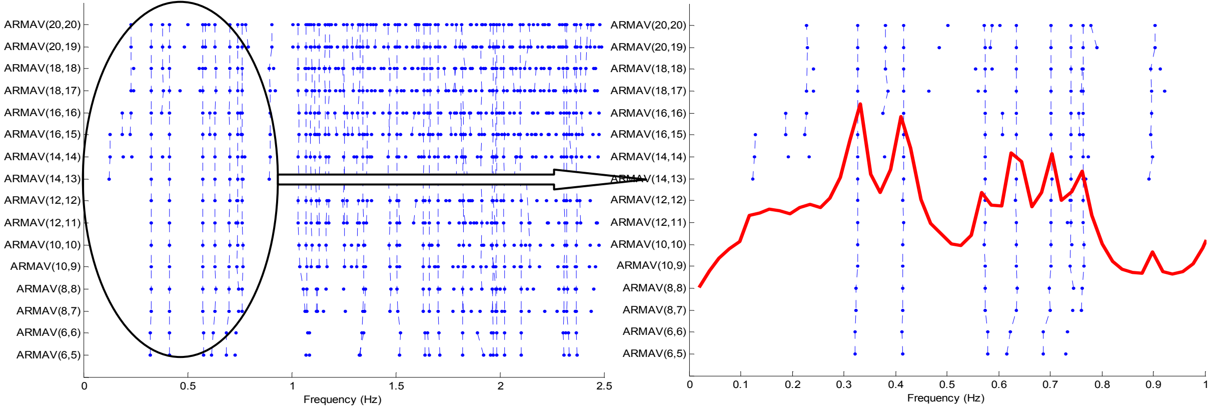
where N is the number of samples and $\upsilon = (na+nb)\cdot m^2$ is the total number of estimated parameters. It is noted that minimizing the FPE is equivalent to minimizing the AIC, provided a large N and low model orders. This effect is shown subsequently.

5 MODAL IDENTIFICATION OF THE EMERSON BRIDGE

The identified modal parameters of the Emerson Bridge are discussed in this section. To demonstrate the accuracy of the results they are compared to the modal parameters of the FEM model. The accuracy of the ARMAV model is also considered herein.

The first objective of system identification has always been successful identification of the system's natural frequencies. The ARMAV model used in this paper considers the model order $(2s,2s-1)$ or $(2s,2s)$, which has been reported to be a better choice for discrete model (Bodeux & Golinval 2001). For this paper, orders of (6, 5), (6, 6), (8, 7), (8, 8)… (20, 19), (20, 20) are used in constructing ARMAV models.

The stability diagram (see Fig. 3) is employed to identify the 'physical' structural modes by varying the model orders. Identification of modal frequencies below 1 Hz is the objectives of this preliminary work. An averaged power spectra density (PSD) function over all of the 16 channels is shown in the same figure to demonstrate that stable modes are indeed located at the peaks of PSD function. Clearly, the first five modes show good agreement. The identified frequencies by ARMAV, PSD and FEM are listed in Table 1. The 4th mode of the FEM model is a mode which combines torsional and lateral motions, and is not identified in this preliminary analysis using the channels recording vertical motion only. The counterpart of 4th mode of ARMAV is the 5th mode of FEM model, which is further demonstrated by the comparison of mode shapes in next section.



(a)Frequency range <2.5 Hz

(b) Frequency range <1 Hz (The curve indicates the average PSD over all the channels)

Figure 3. Stability Diagram

Table 1. Modal Frequencies comparison

Modal Freq.*	ARMAV	PSD	FEM
1	0.3264	0.332	0.2884
2	0.4152	0.4102	0.3849
3	0.5737	0.5664	0.4381
4	0.6329	0.625	0.5993 (5th mode)
5	0.7009	0.7031	0.6635 (6th mode)

*Identified frequencies are less than 1 Hz.

To consider the best order of the model to use for identification, the FPE and AIC are shown in Figure 4, where the trends clearly indicate the fact that a higher modal order results in a better ARMAV model. From the AIC criterion, this indicates that ARMAV models with order $\left(2s,2s-1\right)$ give less error than the models with order $\left(2s,2s\right)$.

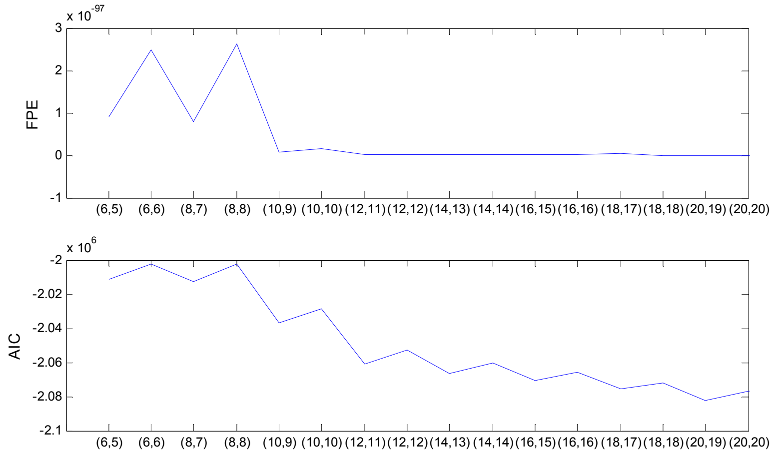


Figure 4. FPE and AIC criteria

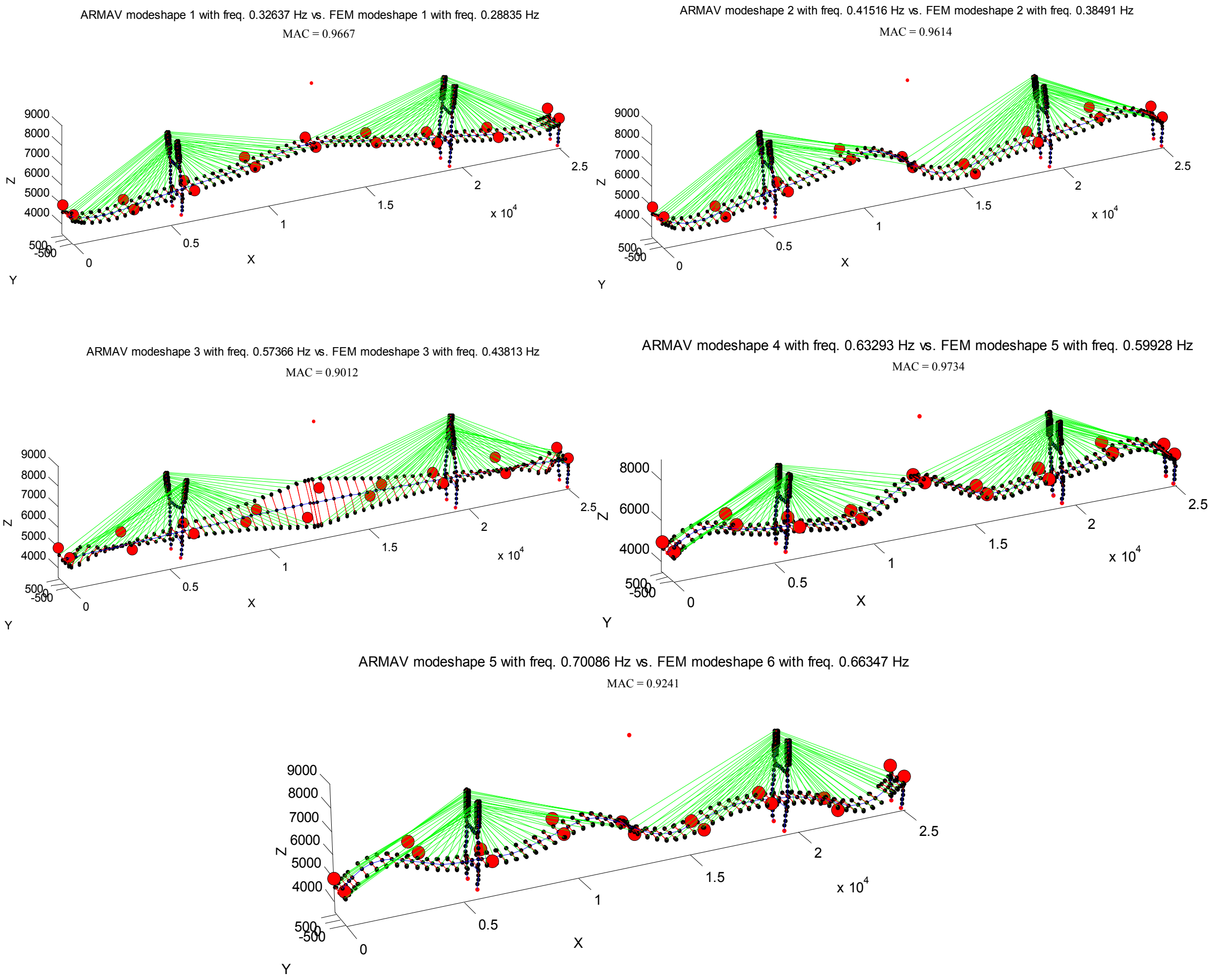


Figure.5 Mode shapes comparison (dots denote the mode shape calculated by ARMAV)

Next, we examine the identified mode shapes and compared them with those of the finite element model. In this section, the first five vertical mode shapes obtained from FEM are compared with those from the ARMAV. The numerically obtained mode shapes are shown in Figure 5 and the identified modes are superimposed on these diagrams with MAC value indicated. Note that there is excellent agreement between the identified and numerical mode shapes, and all the MAC values are greater than 0.9. Please also be aware that the 4th and 5th modes of ARMAV are compared with the 5th and 6th modes of FEM, respectively. The results provide a convincing demonstration of the effectiveness of the ARMAV approach.

6 CONCLUSIONS

This paper presents modal identification results of the Emerson Bridge (Cape Girardeau, Missouri) data using the ARMAV technique. Sixteen, 30-minute vertical acceleration records are employed for identification. Five frequencies and mode shapes are identified and validated through a comparison with an existing FEM of the bridge. The values obtained visually from the PSD are compared with ARMAV. Also, FPE and AIC criteria have been used to evaluate the accuracy of the ARMAV models. Based on the comparisons of natural frequencies and corresponding mode shapes, the ARMAV approach shows great potential in modal identification of long-span bridges. Further study will be carried out based on more data channels and concentrated on identification of coupled and closely-spaced vibrational modes.

7 ACKNOWLEDGEMENTS

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