An Effective Mass Measure for Selecting Free-Free Target Modes

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ABSTRACT

One of the most important tasks in pretest analysis and modal survey planning is the selection of target modes. The target modes are those mode shapes that are determined to be dynamically important using some definition. While there are many measures of dynamic importance, one of the measures that has been of greatest interest to structural dynamicists, is the contribution of each mode to the dynamic loads at an interface. Dynamically important modes contribute significantly to the interface loads and must be retained in any reduced analytical representation. These modes must be identified during a ground vibration test to validate the corresponding finite element model. Structural dynamicists have used interface load based effective mass measures to efficiently identify target modes for constrained structures. The advantage of these measures of dynamic importance is that they are absolute, in contrast to other measures that only indicate the importance of one mode shape relative to another. However, in many situations, especially in aerospace applications, structures must be tested in a free-free configuration. In the case of free-free elastic modes, the effective mass values are zero, making them useless measures of dynamic importance. This paper presents a new effective mass like measure of absolute dynamic importance that can be applied to free-free structures. Numerical examples will be presented to illustrate the application of the new technique and how it relates to other measures such as approximate balanced singular values.

INTRODUCTION

One of the most important tasks in pretest analysis and modal survey planning is the selection of target modes. The target modes are those mode shapes that are determined to be dynamically important using some definition. While there are many measures of dynamic importance [1-7], one of the measures that is of greatest interest to a structural dynamicist, is the contribution of each mode shape to the dynamic loads at an interface. The interface can represent a connection to ground, a connection of a substructure to the rest of a system, or locations where external forces are applied. Dynamically important modes contribute significantly to the interface loads and must be retained in any reduced analytical representation of the substructure. Therefore, these modes must be identified during a ground vibration test to validate the corresponding finite element model.

Modal participation factors [8] have long been used by civil engineers to determine the modal response of buildings to ground motion from earthquakes, while structural dynamicists have extensively used Effective Mass [8] to determine dynamic importance. The advantage of Effective Mass is that it is an absolute measure of dynamic importance in contrast to other measures that only indicate the importance of one mode shape with respect to another. The advantage of an absolute measure is that the original modal representation of the FEM can be truncated such that a desired fraction of the total Effective Mass is retained, guaranteeing that the reduced representation of the substructure will be accurate. A generalization of the Effective Mass measure, called Effective Interface Mass (EIM) [8] is believed to be a more consistent and global measure than the traditional application of Effective Mass. It is also an absolute measure of dynamic importance, but unlike Effective Mass, which determines the contribution of each mode shape to the resultant loads at the interface when the interior of the structure is given a rigid body displacement, the Effective Interface Mass measure determines modal contributions to individual interface loads under more general displacements of the interior of the structure. This provides more dynamically complete results. If the structure is to be tested with the interface degrees of freedom fixed, Effective Mass or Effective Interface Mass can be used to determine the dynamically important modes. If the interface degrees of freedom render the structure statically underdetermined or determinant, Effective Interface Mass reduces to Effective Mass.

However, in many situations, especially in aerospace applications, the structure must be tested in a free-free configuration. In the case of free-free elastic modes, the Effective Mass and Effective Interface Mass values are zero, making either of them useless as a measure of dynamic importance. This paper emulates the development of the Effective Interface Mass measure to formulate a measure of dynamic importance that can be applied to a free-free structure. Numerical examples will be presented to illustrate the application of the new technique.

THEORY

Effective Interface Mass

Initially, the development of the EIM measure of dynamic importance will be discussed to motivate the new free-free measure. The FEM representation of the unconstrained substructure to be analyzed can be written in the form

$$\begin{bmatrix} M_{oo} & M_{oa} \\ M_{ao} & M_{aa} \end{bmatrix} \begin{Bmatrix} \ddot{u}_o \\ \ddot{u}_a \end{Bmatrix} + \begin{bmatrix} K_{oo} & K_{oa} \\ K_{ao} & K_{aa} \end{bmatrix} \begin{Bmatrix} u_o \\ u_a \end{Bmatrix} = \begin{Bmatrix} 0 \\ f \end{Bmatrix} \tag{1}$$

where it has been assumed that the damping is light. Subscript a denotes a matrix partition corresponding to the n_a substructure degrees of freedom at the interface with the remaining structure and subscript o denotes n_o degrees of freedom in the substructure interior. For n total degrees of freedom in the FEM, $n_o + n_a = n$. Note that the a-set degrees of freedom would be constrained during a modal test of the substructure. It is assumed that external loads f are applied only at the substructure interface.

The unconstrained FEM equation of motion in (1) can be transformed to an equivalent Craig-Bampton representation [9] using the transformation

$$u = Tu_{CB} \tag{2}$$

Transformation matrix T is given by

$$T = \begin{bmatrix} \Phi_c & \Psi \\ 0 & I_a \end{bmatrix} \tag{3}$$

in which Φ_c is an $n_o x n_o$ matrix of fixed interface mode shapes satisfying the *o*-set eigenvalue problem, 0 is an $n_a x n_o$ null matrix, Y is an $n_o x n_a$ matrix of shapes giving the static *o*-set displacement of the substructure for a unit displacement of the *a*-set of the form

$$\Psi = -K_{oo}^{-1} K_{oa} \tag{4}$$

and I_a is an identity matrix of order n_a. The first n_o columns of the transformation matrix are fixed interface modes representing the dynamics of the interior of the substructure with respect to the interface. These modes would be measured during a fixed interface modal survey of the substructure. The corresponding eigenvalues are represented by the diagonal matrix Ω_c. The last n_a columns of *T* represent the static response of the substructure to unit displacements of the interface. These columns are called constraint modes in the literature. The CB substructure representation is given by

$$\begin{bmatrix} I & \Phi_c^T \left(M_{oo}\Psi + M_{oa}\right) \\ \left(M_{ao} + \Psi^T M_{oo}\right)\Phi_c & M_{aa} + M_{ao}\Psi + \Psi^T M_{oa} + \Psi^T M_{oo}\Psi \end{bmatrix} \begin{Bmatrix} \ddot{q}_c \\ \ddot{u}_a \end{Bmatrix}$$

$$+ \begin{bmatrix} \Omega_c & 0 \\ 0 & K_{aa} + K_{ao}\Psi \end{bmatrix} \begin{Bmatrix} q_c \\ u_a \end{Bmatrix} = \begin{Bmatrix} 0 \\ f \end{Bmatrix} \tag{5}$$

where the fixed interface modes Φ_c are normalized with respect to the *o*-set mass matrix and q_c represents the corresponding generalized modal coordinates.

The EIM measure is developed by considering the partition of Eq. (5) that governs the response of the fixed interface modal coordinates given by

$$\ddot{q}_c + \Omega_c q_c = P_c \ddot{u}_a \tag{6}$$

The right hand term represents a forcing function that excites the fixed interface modes due to motion of the interface degrees of freedom. The coefficient matrix that multiplies the interface acceleration inputs is defined as

$$P_c = -\Phi_c^T \left[M_{oo}\Psi + M_{oa}\right] = \begin{bmatrix} P_{c1} \\ P_{c2} \\ \vdots \\ P_{cn_o} \end{bmatrix} \tag{7}$$

The $n_o x n_a$ matrix P_c is called the *Modal Participation Factor* matrix. The *i*th row P_{ci} contains the modal participation factors for the *i*th fixed interface mode. They represent multiplication factors for the acceleration inputs from the interface degrees of freedom. The larger the *j*th entry in P_{ci}, the more the *i*th mode will be excited by the *j*th input.

It was shown in Reference [8] that the trace of the $n_a x n_a$ matrix $\overline{M}_i = P_{ci}^T P_{ci}$ gives a measure of the contribution of the *i*th fixed interface mode to the loads at the interface. Summing the contributions from all the modes produces the $n_a x n_a$ reduced interior mass (RIM) matrix

$$\overline{M} = \sum_{i=1}^{n_o} \overline{M}_i = \sum_{i=1}^{n_o} P_{ci}^T P_{ci} = P_c^T P_c \tag{8}$$

A measure of the completeness and accuracy of the interface loads predicted by a reduced analytical representation, where only n_t and not all of the n_o fixed interface modes are retained, can be determined by comparing the predicted matrix $\overline{M}_t$ relative to the complete matrix $\overline{M}$ using the trace norm. In addition, a measure of the dynamic importance of each fixed interface mode can be determined by comparing $\overline{M}_i$ with $\overline{M}$. The closed form relation for the reduced interior mass matrix was shown to be of the form

$$\overline{M} = M_{ao}\Psi + \Psi^T M_{oa} + \Psi^T M_{oo}\Psi + M_{ao}M_{oo}^{-1}M_{oa} \tag{9}$$

This expression can be computed based solely upon various partitions of the FEM mass and stiffness matrices and is thus totally independent of any eigenvalue solution. Therefore, it can be used as an absolute reference with respect to which the dynamic importance of each mode shape can be computed. The EIM measure of dynamic importance computes the fractional contribution of each fixed interface mode to the trace of the RIM matrix. The EIM value of the ith mode is given by

$$E_{Ii} = \frac{tr\left(\overline{M}_i\right)}{tr\left(\overline{M}\right)} \tag{10}$$

Equation (10) assumes that translational and rotational interface degrees of freedom are weighted equally. Effective Interface Mass gives an absolute measure of dynamic importance for modes that are constrained by an interface such that there is no rigid body motion. Free-free modes have zero Effective Interface Mass.

Free Effective Mass

In the case of a free-free structure, assume now that the a-set in Eq. (1) contains all degrees of freedom where there are external loads applied, as in a vibration test. The o-set is its compliment. In the spirit of the development of EIM, the equations of motion for the free-free structure in Eq. (1) can be transformed to free-free modal coordinates in the form

$$\ddot{q} + \Omega q = \phi_a^T f \tag{11}$$

where q are the modal coordinates, Ω is a diagonal matrix of corresponding eigenvalues, ϕ_a are the free-free modes shapes partitioned to the force input locations, and f are the corresponding excitation forces. The modes are assumed to be mass normalized.

In the EIM computation, the dynamic importance of the fixed interface modes was based on the coefficient matrix P_c in the forcing term in the corresponding modal equation of motion, Eq. (6). In an analogous manner, a measure of dynamic importance for the free-free modes can be based on the forcing coefficient matrix $P_f = \phi_a^T$ in Eq. (11). The ith row of P_f contains the participation factors for the ith free-free mode. The larger the jth entry in P_{fi}, the more the ith mode is excited by the jth input. Analogous to the RIM matrix used in the EIM method, a corresponding matrix can be generated in the free-free case in the form

$$D = \sum_{i=1}^{n} D_i = \sum_{i=1}^{n} P_{fi}^T P_{fi} = P_f^T P_f = \phi_a \phi_a^T \tag{12}$$

The significance of the matrix D can be illustrated by pre-multiplying Eq. (11) by ϕ_a and rearranging to produce the output equation

$$y = \phi_a \ddot{q} = -\phi_a \Omega q + \phi_a \phi_a^T f = -\phi_a \Omega q + Df \tag{13}$$

in which y is the acceleration at the input locations and D is recognized as the direct throughput matrix.

Equation (13) indicates that modes that are strongly excited by the inputs contribute strongly to the acceleration response at the input locations, as would be expected. In general, there are many more modes than can ever be computed and used in a numerical simulation. The dynamic completeness of the truncated system with respect to the effect of the applied loads can be determined by comparing the direct throughput matrix computed using the truncated set of modes given by

$$D_t = \sum_{i=1}^{n_m} D_i = \sum_{i=1}^{n_m} \phi_{ai}\phi_{ai}^T \tag{14}$$

with the complete D matrix using an appropriate norm, such as the trace. In general, the rigid body modes are not of interest, therefore, D_t should be compared with D_e corresponding to the complete set of elastic modes. The proposed Free Effective Mass (EMf) measure for the ith elastic mode is then given by the expression

$$E_{fi} = \frac{tr(D_i)}{tr(D_e)} \tag{15}$$

analogous to the measure for EIM.

The strength of the EIM measure of dynamic importance is that it is absolute. The RIM matrix $\overline{M}$ can be computed without having to compute all of the fixed interface modes, as shown in Eq. (9). In order to have the Free Effective Mass measure also be absolute, a closed form expression must be found for the reference matrix

$$D_e = \phi_{ea}\phi_{ea}^T \tag{16}$$

where ϕ_{ea} represents the complete set of elastic modes partitioned to the input locations. Assuming that the free-free modes are mass normalized, it can be shown that

(17)

in which ϕ_r are the rigid body modes, ϕ_e are the elastic modes, and M is the analytical mass matrix. The term $\phi_r\phi_r^T M = Q_r$ is an easily computed oblique projector [10] onto the rigid body space of the system. Rearranging Eq. (17) and dividing matrices into a-set and o-set partitions produces the expression

$$\begin{bmatrix} \phi_{oe}\phi_{oe}^T & \phi_{oe}\phi_{ae}^T \\ \phi_{ae}\phi_{oe}^T & D_e \end{bmatrix} \begin{bmatrix} M_{oo} & M_{oa} \\ M_{ao} & M_{aa} \end{bmatrix} = \begin{bmatrix} I_{oo} - Q_{roo} & -Q_{roa} \\ -Q_{rao} & I_{aa} - Q_{raa} \end{bmatrix} \tag{18}$$

where the expression in Eq. (16) has been used. The two equations involving D_e are given by

$$\phi_{ae}\phi_{oe}^T M_{oo} + D_e M_{ao} = -Q_{rao} \tag{19}$$

$$\phi_{ae}\phi_{oe}^T M_{oa} + D_e M_{aa} = I_{aa} - Q_{raa} \tag{20}$$

These equations can be solved to produce the result

$$D_e = \left[I_{aa} - Q_{raa} + Q_{rao}M_{oo}^{-1}M_{oa}\right]\left[M_{aa} - M_{ao}M_{oo}^{-1}M_{oa}\right]^{-1} \tag{21}$$

Note that D_e has units of inverse mass. It may also be of interest to determine the contribution of each free-free mode with respect to each input location. This can be accomplished using the expression

$$e_f = \left[P_f\right]^{\wedge 2}\left[d_e\right]^{-1} \tag{22}$$

in which $^{\wedge 2}$ denotes a term-by-term square, and $[d_e]$ is a matrix containing the diagonal terms in D_e. The term e_{fij} gives the fractional contribution of the ith mode to the total influence of the jth input on the structure.

Relationship to Approximate Balanced Singular Values

The equations of motion governing the response of the free-free modes given in Eq. (11) can be generalized to include damping such that the equation for the ith mode is given by

$$\ddot{q}_i + 2\varsigma_i \omega_i \dot{q}_i + \omega_i^2 q_i = P_{fi} f \qquad (23)$$

in which ς_i is the corresponding damping coefficient. It is assumed that damping is small and the modal frequencies are well separated. A general acceleration output equation will be assumed in the form

$$y = C_a \ddot{q} \qquad (24)$$

in which C_a is the corresponding output influence matrix. A standard method of model reduction that is used extensively in control system design and analysis to identify important modes is Moore's method of internal balancing [2]. Assuming that the system is stable, a controllability grammian W_C and an observability grammian W_O can be defined as

$$W_c = \int_0^\infty e^{At} B B^T e^{A^T t} dt \qquad W_o = \int_0^\infty e^{A^T t} C^T C e^{At} dt \qquad (25)$$

where A is the state matrix, and B and C represent the input and output influence matrices, respectively, from a standard state-space representation. Moore's method is based on the fact that there exists a transformation of the state vector, S, such that the transformed controllability and observability grammians are equal and diagonal

$$\overline{W_c} = S^{-1} W_c S = \overline{W_o} = S^{-1} W_o S = \sigma^2 \qquad (26)$$

The ith entry σ_i^2 is called the balanced singular value for the ith balanced state. It has been shown that the minimum energy input required to reach a unit value in the ith state is given by $1/\sigma_i^2$. States with large balanced singular values can be excited by a small amount of input energy, resulting in large output energy. These states are also more controllable and observable than states with small balanced singular values. An accurate reduced representation of the system can be obtained by eliminating states with small balanced singular values.

In general, it is difficult to apply Moore's method directly to a FEM representation. However, it has been demonstrated that Moore's internally balanced representation asymptotically approaches the standard modal representation as damping goes to zero [5]. Therefore, it can be shown that an approximate balanced singular value for the ith mode in Eqs. (23) and (24) is given by

$$\tilde{\sigma}_i^2 = \frac{\sqrt{C_{ai}^T C_{ai}} \sqrt{P_{fi} P_{fi}^T}}{4\varsigma_i} \qquad (27)$$

in which C_{ai} is the ith column of the output influence matrix. Blelloch [7] showed that the approximate balanced singular value of a mode is its peak contribution to the maximum singular value of the transfer function matrix. Selecting free-free modes based upon their approximate balanced singular values therefore minimizes the peak magnitude of the error in the corresponding transfer function matrix. It is important to note that the approximate balanced singular value is a relative measure of the dynamic importance of one mode with respect to another. The absolute measure of a mode's dynamic importance can only be determined by computing all the modes to calculate the peak value of the maximum singular value of the transfer function matrix.

For the case of modal acceleration output, the output influence parameter in Eq. (23) has the form

$$C_{ai} = \{0 \quad \cdots \quad 1_i \quad 0 \quad \cdots\}^T \qquad (28)$$

The approximate balanced singular value is then given by

$$\tilde{\sigma}_i^2 = \frac{\sqrt{\phi_{ai} \phi_{ai}^T}}{4\varsigma_i} \qquad (29)$$

Using the definitions in Eq. (14) and (15), the square of the approximate balanced singular value is given by

$$\left(\tilde{\sigma}_i^2\right)^2 = \frac{tr(D_i)}{16\varsigma_i^2} = \frac{tr(D_e)}{16\varsigma_i^2} E_{fi} \tag{30}$$

In structural loads analysis, it is common practice to assume that the damping is approximately the same for the all modes of interest. The square of the *i*th approximate balanced singular value is then directly proportional to the EMf value for the *i*th mode shape. Therefore, ranking the modes based upon the EMf measure is equivalent to ranking the modes based upon the square of the approximate balanced singular values for modal acceleration output. If accelerometers are placed at the input locations, the output influence parameter is given by $C_{ai} = \phi_{ai}$. The approximate balanced singular value is then

$$\tilde{\sigma}_i^2 = \frac{tr(D_i)}{4\varsigma_i} = \frac{tr(D_e)}{4\varsigma_i} E_{fi} \tag{31}$$

which is directly proportional to the EMf measure. Ranking the modes based upon EMf is then equivalent to ranking the modes based upon the approximate balanced singular values for acceleration output at the input locations. The big advantage of the EMf measure is that it is absolute, while in general, the approximate balanced singular values cannot be made absolute without computing all of the structure modes which is in general prohibitive.

NUMERICAL EXAMPLES

Simple Beam

A simple free-free Euler beam is considered in plane motion for an initial application of the proposed Free Effective Mass mode ranking technique. The unrestrained beam will experience excitations at two transverse locations; one at the end and the other at the one-third length position. The excitations can be due to external forces during operation, actuators for control, or perhaps actuators during a modal vibration test. A finite element model was constructed containing 22 nodes and 21 elements. The model was statically reduced to the 22 transverse degrees of freedom at each of the nodes. The reduced system then possesses 22 modes; two rigid body modes and 20 elastic modes. System parameters were such that the fundamental mode is at 4.89 Hz. and the highest mode is at 665.09 Hz. Because the system is unconstrained, the usual Effective Mass or Effective Interface Mass cannot be used to rank the modes.

Equations (15) and (21) were used to compute the Free Effective Mass for the 20 elastic mode shapes. Table 1 lists the modes in descending order from greatest to least dynamic importance. Mode two is the most dynamically important relative to the input locations, mode 18 is the least important. The cumulative sum column in the table indicates that fifteen of the modes comprise 91% of the dynamics excited by the inputs. Assuming that there are accelerometers at the input locations and 0.1% modal damping, Figure 1 illustrates the maximum singular value of the corresponding frequency response matrix with no direct throughput versus frequency in a band covering the first 11 modes. Each peak represents one of the modes. The EMf modal ranking for the top 11 modes from Table 1 is also shown in the figure. As predicted, the EMf ranking picks off peaks in descending order of magnitude which is consistent with an approximate balanced singular value modal ranking. Considering all 20 elastic modes, the approximate balanced singular value gives the same ranking as EMf, as indicated in Table 1. A corresponding reduced representation based upon EMf will then accurately predict the peak value in the frequency response. Note that very small levels of damping are used in this example and the next because the modal representation only approaches Moore's internally balanced representation as the damping goes to zero.

Generic Spacecraft

A second more complicated example considers the attitude control of a generic communications satellite. The spacecraft finite element model, containing 1191 nodes and 1262 elements, is illustrated in Fig. 2. The satellite is to be controlled using 6 actuators distributed on the main bus as shown in Fig. 2. Numbers indicate the translations that are constrained at each of the actuator locations. In order to validate the finite element model, a modal survey would be conducted with the spacecraft supported by a soft suspension and excited at the controller locations. There are 19 elastic modes in the frequency range between 0.0 and 17.0 Hz. Figure 3. illustrates a typical mode shape.

The Free Effective Mass was computed to rank the 19 modes in the designated frequency range. Table 2 lists the modes sorted by their EMf values. Mode 3 is the most dynamically important, while mode 17 is least important.

Note that the EMf values are small, even for the most dynamically important modes. The Effective Interface Mass measure is in general dominated by the lower frequency modes. They contribute most strongly to the interface loads. In contrast, EMf tends to distribute the dynamic importance over a larger set of modes. In practice, 90% Effective Interface Mass is generally considered an acceptable level of dynamic completeness. Further work must be performed to determine an acceptable level of dynamic completeness for EMf that would produce a reduced model of acceptable size.

Assuming that there are accelerometers at the input locations and 0.05% modal damping, Figure 4 shows the maximum singular value of the corresponding frequency response matrix with no direct throughput versus frequency in a band from 0.0 to 12.0 Hz. Each peak represents one of the modes. The EMf modal ranking for each of the modes from Table 4 is also shown in the figure. Note that only 10 peaks appear corresponding to 10 of the total 18 modes in the frequency band. A peak for the 19^{th} mode at 16.48 Hz. is also absent from an expanded plot. This indicates that 9 of the modes are not observable/controllable. Therefore, in this example, the approximate balanced singular values, and thus also EMf, is not a direct measure of the observability and controllability of the system through the input locations. But, EMf is still a measure of the contribution of each of the free-free modes to the response at the input locations and thus a measure of the dynamic completeness of a reduced system with respect to the effect of the inputs. If the 10 observable/controllable modes are ranked by maximum singular value, the order corresponds to that predicted by EMf, as shown in Table 2. The tenth ranked mode based on EMf, mode number 2, is not observable/controllable. But if the point on the curve in Fig. 4 at the frequency corresponding to mode 2 were selected, the resulting singular value would rank tenth compared to the peak values listed in Table 2.

CONCLUSION

A new method has been presented for selecting target modes for either finite element model reduction or pretest analysis of a modal vibration test. The state-of-the practice in the structural dynamics community is to use measures of dynamic importance such as Effective Mass or Effective Interface Mass. However, these methods only work for constrained structures. In many situations, especially in the aerospace industry, the structure will operate in a free-free configuration. The formulation of the new method emulates the Effective Interface Mass approach, but it can be applied to free-free modes. The Free Effective Mass measure of dynamic importance is based upon a mode's contribution to the direct throughput matrix from the standard linear state-space representation. It was shown that EMf is equivalent to ranking the free-free modes based upon their approximate balanced singular values. The EMf value of the *i*th mode thus corresponds to the fractional contribution of the mode to the peak in the frequency response for the acceleration response at the input locations. In contrast to the general case of approximate balanced singular values, EMf gives the absolute measure of the dynamic importance of the modes. A truncated set of mode shapes can therefore be checked for dynamic completeness with respect to the response at the input locations. This gives a direct measure of how much each mode is influenced by the inputs. Two examples were considered. The first was a simple free-free beam. The EMf value was computed and used to rank the dynamic importance of each of the 20 elastic modes. The maximum singular value for the frequency response function matrix for collocated inputs and outputs was then computed versus frequency. Each of the 20 modes was observable and controllable, and thus appeared as a peak in the corresponding plot. Picking off peaks in descending order produced the same dynamic importance ranking as the EMf approach, as predicted. The second example considered a more complex model of a generic satellite system. The same analysis was performed. Although some of the modes were not observable/controllable, the EMf ranking and the maximum singular value ranking produced the same results.

It is believed that Free Effective Mass can be used to efficiently rank the dynamic importance of free-free mode shapes. In contrast with other effective mass measures, it was shown that EMf tends to distribute the dynamic importance over a larger set of modes. Future work will be performed to determine an acceptable level of dynamic completeness for EMf that will produce reduced models of acceptable size.

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Table 1. Mode ranking for simple beam.

Mode	Freq. (Hz)	EMf	CumSum	Mode	MaxSV(db)
2.00	13.41	0.0963	0.0963	2	44.8585
5.00	64.03	0.0856	0.1819	5	43.9528
6.00	88.99	0.0844	0.2664	6	43.7264
3.00	26.17	0.0802	0.3465	3	43.3302
9.00	187.43	0.0742	0.4207	9	42.5943
1.00	4.89	0.0721	0.4927	1	42.4245
4.00	43.06	0.0635	0.5563	4	41.3491
8.00	150.73	0.0590	0.6152	8	40.6698
7.00	117.93	0.0533	0.6685	7	39.7642
12.00	317.96	0.0531	0.7216	12	39.7075
10.00	227.65	0.0467	0.7683	10	38.4623
13.00	367.62	0.0424	0.8107	13	37.7830
16.00	521.97	0.0397	0.8503	16	37.2170
19.00	644.06	0.0338	0.8841	19	36.3113
11.00	271.40	0.0321	0.9162	11	35.5755
15.00	471.49	0.0283	0.9445	15	34.5566
20.00	665.09	0.0218	0.9664	20	34.1604
14.00	418.92	0.0141	0.9804	14	28.7830
17.00	569.65	0.0115	0.9919	17	27.5377
18.00	610.42	0.0081	1.0000	18	26.5189

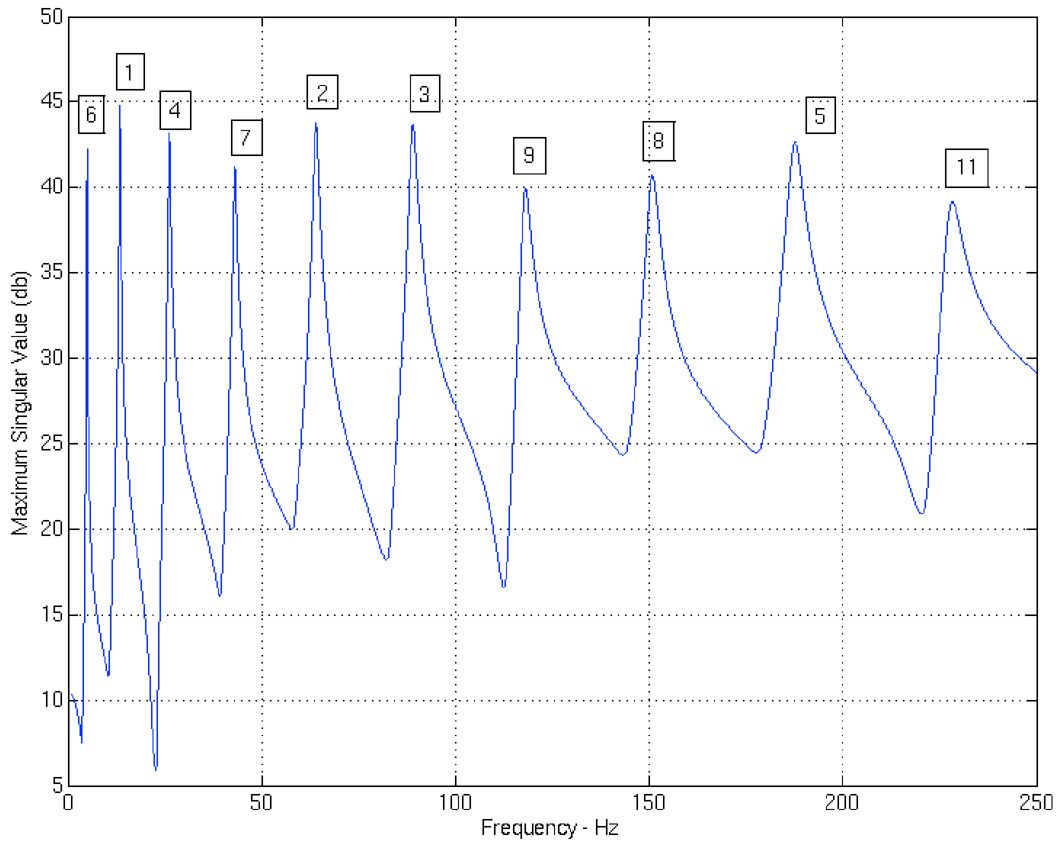

Fig. 2 Maximum singular value for frequency response matrix and EMf ranking for simple beam.

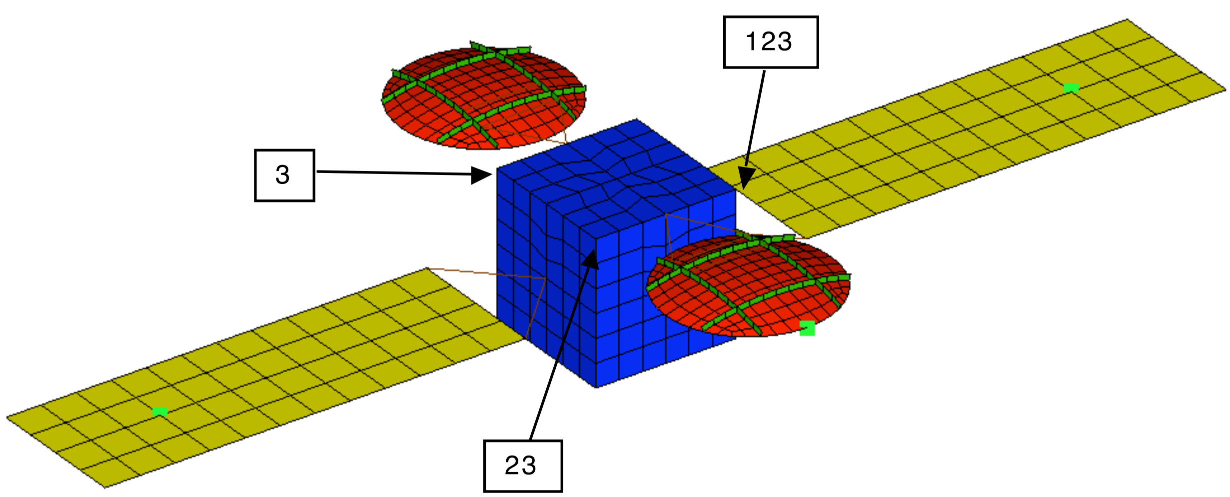

Fig. 2. Generic spacecraft finite element model.

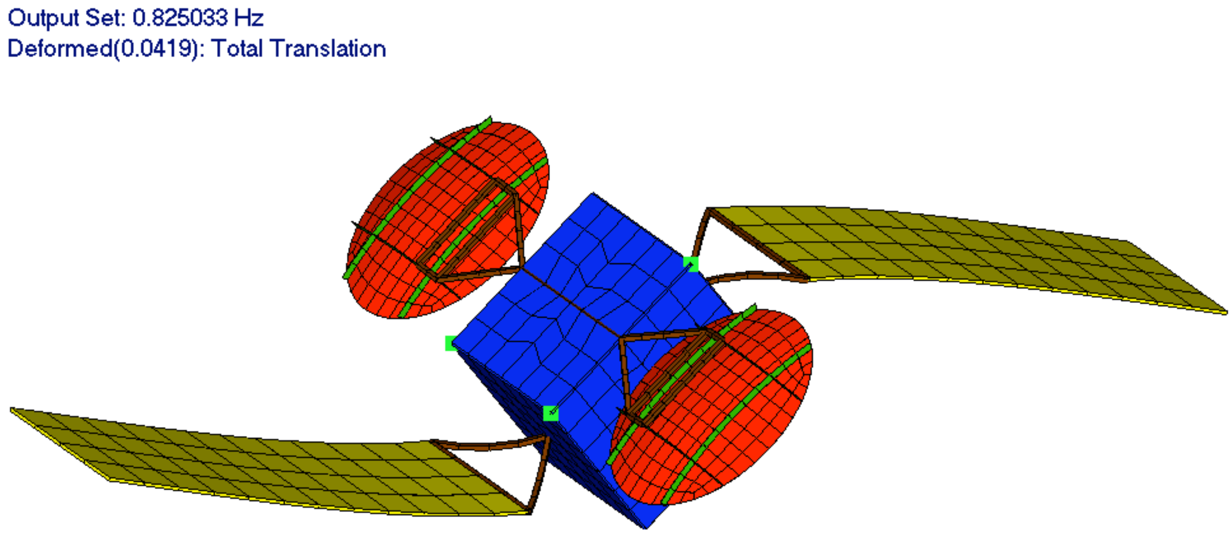

Fig. 3. Free-free elastic mode 3 – 0.83 Hz.

Table 2. Mode ranking for generic spacecraft.

Mode	Frequency (Hz)	EMf	MaxSV (db)
3	0.83	6.8593e-04	-15.20
4	1.84	4.8755e-04	-18.04
12	6.61	1.1861e-04	-30.53
8	3.77	1.1550e-04	-30.72
10	4.37	9.7532e-05	-32.42
6	2.91	4.3450e-05	-39.42
16	9.93	2.5902e-05	-43.58
1	0.31	2.3998e-05	-44.53
11	6.10	1.9032e-05	-45.85
2	0.64	9.2813e-06	
14	7.69	8.8666e-06	-51.15
7	3.56	5.7827e-06	
13	7.68	4.6725e-06	
9	4.11	2.3858e-06	
15	9.83	1.3793e-06	
18	11.99	8.4887e-07	
5	2.79	7.5507e-09	
19	16.48	1.2357e-10	
17	11.99	4.1950e-11	

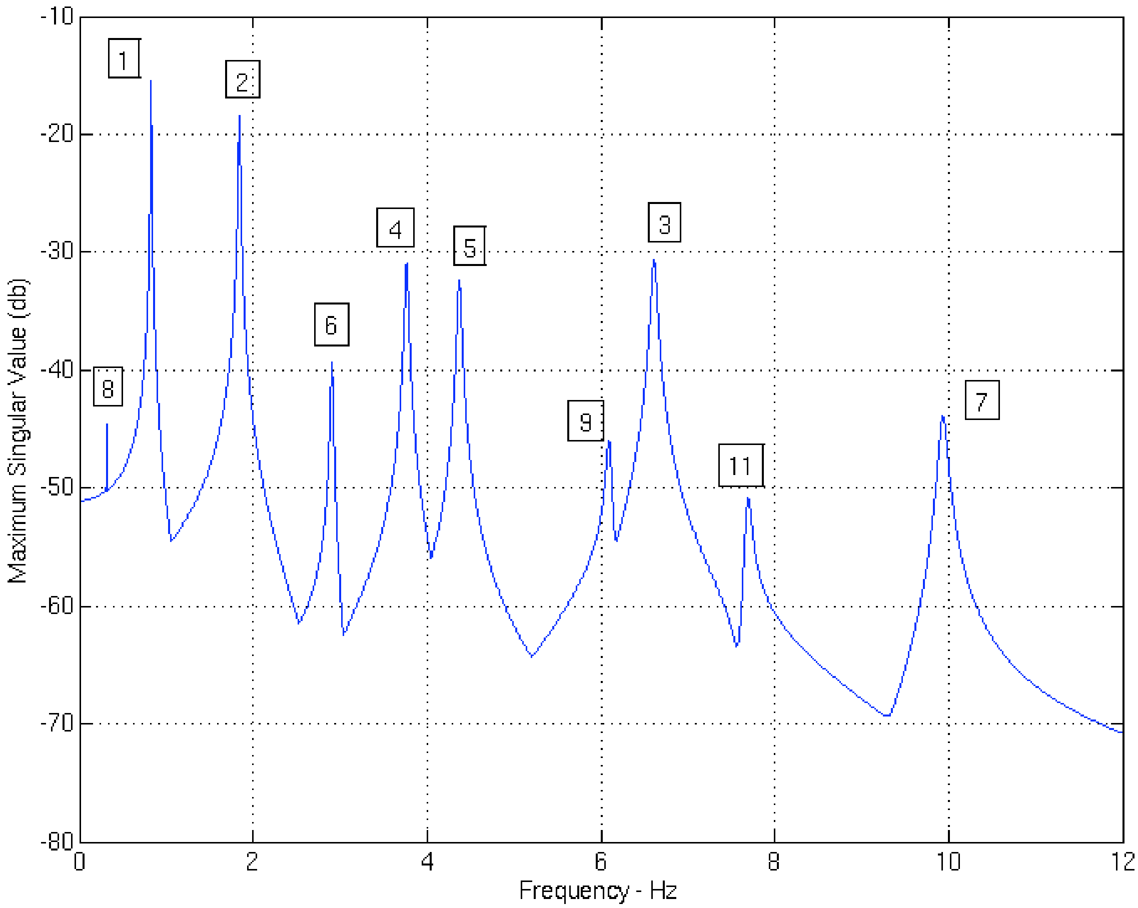

Fig. 4. Maximum singular value for frequency response matrix and EMf ranking for generic spacecraft.